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# Mass outflow rate from relativistic matter quasi-spherically accreting onto black holes

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**Abstract** We compute mass outflow rate $R_m$ from relativistic matter accreting quasi-spherically onto Schwarzschild black holes Taking the pair-plasma pressure mediated shock surface as the *effective* boundary layer (of the black hole) from where bulk of the outflow is assumed to be generated, computation of this rate is done using combinations of exact transonic inflow and outflow solutions We find that $R_m$ depends on the initial parameters of the flow, the polytropic index of matter, the degree of compression of matter near the shock surface and on the location of the shock surface itself We thus not only study the variation of the mass outflow rate as a function of various physical parameters governing the problem but also provide a sufficiently plausible estimation of this rate

**Keywords** Astrophysical black hole, accretion and outflow, AGN and quasars

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Computation of mass outflow rates from the advective accretion disks around black holes and neutron stars has very recently been done [1-4] by self consistently combining the exact transonic accretion and wind solutions. Rigorously justifying the fact that most of the outflowing matter comes out from the CENtrifugal Pressure Supported BOundary Layer (CENBOL) it has been shown there that the Rankine-Hugoniot shock location or the location of the maximum polytropic pressure acts as the CENBOL. However, for some black hole models of active galactic nuclei, inflow may not have accretion disk ([5] and references therein). Accretion is then quasi-spherical having almost zero or negligible angular momentum (Bondi [6] type accretion) and the shock is not of Rankine-Hugoniot type. On the otherhand; absence of angular momentum rules out the possibility of formation of the polytropic pressure maxima. For this type of accretion, absence of intrinsic angular momentum of the accreting material does not allow CENBOL formation. It has been shown that [7-8] for quasi-spherical accretion onto black holes, steady state situation may be developed where a standing collisionless shock may

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form due to the plasma instabilities and for nonlinearity introduced by small density perturbation. This is because, after crossing the sonic point the infalling matter (in plasma form) becomes highly supersonic. Any small perturbation and slowing down of the infall velocity will create a piston and produce a shock. A spherically symmetric shock produced in such a way will accelerate a fraction of the inflowing plasma to relativistic energies. The shock accelerated relativistic particles suffer essentially no Compton loss and are assumed to lose energy only through $p$–$p$ collision. These relativistic hadrons are not readily captured by the balck hole [9] rather considerable high energy density of these relativistic protons would be maintained to support a standing, collisionless, spherical shock around the black hole (see [8] and references therein). Thus, a self-supported standing shock may be produced *even* for accretion with zero angular momentum.

This type of shock formation has a *very important* consequence regarding the formation and dynamics of mass outflow from relativistic matter accreting with zero intrinsic angular momentum (*no* accretion disk) onto black holes which we explore in this *rapid communication*. As there was no such attempt available in the literature which computes the mass loss rate from zero angular momentum quasi-spherical Bondi type accretion, our work, *for the first time* we believe, could shed light on the nature of the outflow from the models of AGNs with no accretion disks. In this work, we take the above mentioned *pair-plasma pressure mediated shock surface* as the alternative of the CENBOL which can be treated as the *effective* physical hard surface which, in principle mimics the ordinary stellar surface regarding the mass outflow

At the shock surface, density of the post-shock material shoots up and velocity falls down, infalling matter starts piling up on the shock surface. The post shock relativistic hadronic pressure then gives a kick to the piled up matter, the result of which is the ejection of outflow from the shock surface. The fraction of energy converted, the shock compression ratio $R_{comp}$ (in the notation of [4]), along with the ratio of post shock relativistic hadronic pressure to infalling ram pressure at a given shock location are obtained from the steady state shock solution of Ellision and Eichler [10, 11]. In this work, we self-consistently calculate the shock location as a function of the specific energy $\varepsilon$ of the infalling matter and accretion rate using the above mentioned quantities. The amount of mass outflow rate $R_m$ from the shock surface is then obtained using combination of *exact* transonic inflow outflow solutions and the dependence of $R_m$ on various physical entities governing the inflow-outflow system has been studied. We thus quantitatively compute the mass outflow rate *only* from the inflow parameters. In this way, we analytically connect the accretion and wind type topologies self-consistently. The condition necessary for the development and maintenance of such a self–supported spherical shock described above is satisfied for the high Mach number solutions [10]. Keeping this in the back of our mind, for our present work, we concentrate only on low energy accretion to obtain high shock Mach number.

Using Paczyn'ski-Wiita [12] potential, for a Schwarzschild type black hole, equations (in dimentionless geometric unit) governing the polytropic inflow are (See [4] and references therein for detail):

a) Conservation of specific energy :

$$\varepsilon = \frac{u(r)^2}{2} + na(r)^2 - \frac{1}{2(r-1)}, \tag{1}$$

b) Mass conservation equation :

$$\dot{M}_{in} = \theta_{in}\rho(r)u(r)r^2 , \tag{2}$$

where $\theta_{in}$ is the solid angle subtended by the inflow. We assume that for our model, the effective thickness of the shock $\Delta_{sh}$ is small enough compared to the shock standoff distance, *i.e.*,

$$\Delta_{sh} << r_{sh} .$$

The shock accelerated relativistic protons produce pions through post-shock inelastic collisions.

$$p + p \rightarrow p + p + \pi^{\pm} + \pi^{0} .$$

Pions generated by this process, decay into relativistic electrons, neutrinos & antinutrinos and produces high energy $\gamma$ rays. These electrons produce the observed non-thermal radiation of AGN by Synchrotron and inverse Compton scattering. The overall efficiency of this mechanism depends largely on the shock location. It has been shown [13] that almost half of the energy flux that goes into relativistic particles is lost due to neutrinos.

At the shock, density of matter will shoot up and inflow velocity fall down abruptly. If $(\rho_-, u_-)$ and $(\rho_+, u_+)$ are the pre and post-shock densities and velocities respectively, then

$$\frac{\rho_+}{\rho_-} = R_{comp} = \frac{u_-}{u_+} , \tag{3}$$

where $R_{comp}$ is the shock compression ratio (in the notation of [4]). For high shock Mach number solution (which is compatible with our low energy accretion model), the expression for $R_{comp}$ can be well approximated as

$$R_{comp} = 1.44\, M_{sh}^{\frac{1}{4}} , \tag{4}$$

where $M_{sh}$ is the shock Mach number and eq. (4) holds good for $M_{sh} \lesssim 4.0$ [11]. However, the scattering mean free path of the relativistic hardrons produced by this process are assumed to be small enough so that they could encounter the full shock compression ratio while crossing the shock.

If we now assume that a fraction $\varepsilon_F$ of the infalling energy is converted into radiation through the hadronic collision $(p-p)$ and mesonic $(\pi^{\pm}, \pi^{0})$ decay, this $\varepsilon_F$ will allow convergent steady state solutions and in the case of quasi-spherical infall, allows the development and maintenance of a standing, collisionless shock at a fixed distance $r_{sh}$ measured in the unit of Scwarzschild radius.

Defining $\delta$ as the ratio of downstream relativistic particle pressure to incoming ram pressure at the shock, and $\sigma_{pp}$ be the collision cross section for relativistic protons, the shock location $r_{sh}$ can easily be expressed as

$$r_{sh} = \frac{3\sigma_{pp} c \dot{M}_{in}}{u_{sh}^{2}\theta_{in}}\left(\frac{\delta}{\varepsilon_F}\right), \tag{5}$$

where $u_{sh}$ stands for the *inflow* velocity at the shock. We choose polytropic outflow with a different polytropic index $\gamma_0 < \gamma$ due to momentum deposition. As a fraction of infalling energy density ($\varepsilon_F$) is converted into radiation, specific energy of the outflow is somewhat less than that of the inflow. Nevertheless, the outflow specific energy is also kept constant throughout the flow.

The following two conservation laws are valid for the outflow :

$$\varepsilon' = \frac{v(r)^2}{2} + n'a(r)^2 - \frac{1}{2(r-1)}, \tag{6}$$

$$\dot{M}_{out} = \Theta_{out}\,\rho(r)v(r)r^2\,, \tag{7}$$

where $\varepsilon'$ is the specific energy of the outflow and $\varepsilon' < \varepsilon$ and $n' = (\gamma_0 - 1)^{-1}$ is the polytropic constant of the outflow. $\Theta_{out}$ is the solid angle subtended by the outflow and $v(r)$ is the radial velocity of the outflow. For simplicity of calculation, we assume that the outflow is also quasi-spherical and $\Theta_{out} \approx \Theta_{in}$.

Defining $R_m$ as the mass outflow rate, we obtain

$$R_m = \frac{\dot{M}_{out}}{\dot{M}_{in}}. \tag{8}$$

It is obvious from the above discussion that $R_m$ should have some complicated functional dependences on the following parameters.

$$R_m = \psi\,(\varepsilon, \dot{M}_{in}, r_{sh}, R_{comp}, \gamma, \gamma_0). \tag{8a}$$

The shock location is calculated by simultaneously solving the eqs. (1), (2) and (5). Using fourth order Runge Kutta method, $u(r)$ and $a(r)$ are computed along the inflow from the *inflow* sonic point till the position where the shock forms. With the known value of $\varepsilon'$ and $\gamma_0$, it is easy to compute the location of the sonic point of the *outflow* from eqs. (6) and (7). Runge-Kutta method is employed to integrate from the *outflow* sonic point towards the black hole to find out the outflow velocity $v$ and density $\rho$ *at the shock location*. The outflow rate is then computed using eqs. (8). Figure 1 shows one of our typical solutions (for a $10 M_\odot$ black hole) which combines the accretion and the outflow. The input parameters are $\varepsilon = 0.001$, $\dot{M}_{in} = 1.0$ Eddington rate ($\varepsilon_d$ stands for the Eddington rate in the figure) and $\gamma = \frac{4}{3}$ corresponding to relativistic inflow. The solid curve with an arrow represents the pre-shock region of the inflow and the solid vertical line with double arrow at $X_{pps}$ (the subscript *pps* stands for *pair* plasma mediated shock) represents the shock transition. Three dotted curves show the three different outflow branches corresponding to different polytropic index of the outflow as $\gamma_0 = 1.3$ (left most curve), 1.275 (middle curve) and 1.25 (rightmost curve). It is evident from the figure that the outflow moves along the solution curves completely different from that of the self-wind solution (solid line marked with an outward directed arrow) of the inflow which passes through the sonic point $P_s$. The mass loss ratio $R_m$ for these cases are 0.0023, 0.00065 and 0.00014 respectively. It is observed that $R_m$ monotonically increases with energy. This is because as $\varepsilon$ increases keeping the Eddington rate of the inflow fixed, the shock Mach number $M_{sh}$ decreases, result of which is the decreament of shock location $r_{sh}$ and post shock density [*via* eq. (4)] but the increment of the post shock fluid velocity ($v_{sh}$) with which the matter leaves the shock surface. The outfow rate $R_m$ which is the product of these three quantities, in general, increases monotonically with $\varepsilon$ due to the combined tug of war of these three quantities. Moreover, closer the shock forms to the black hole, the greater will be the amount of gravitational potential available to be put onto the relativistic hadrons to provide more outward pressure at the shock boundary which gives a stronger kick to the accreting matter, the result of which is the increment in $R_m$. All these points are manifested in Figure 2 where we have shown the variation of $R_m$ as a function of compression ratio $R_{comp}$ (solid curve), the shock location $r_{sh}$ (dotted curve) and the injection velocity of the outflow $v_{sh}$ (dashed curve). The figure is drawn for a fixed $\gamma = \frac{4}{3}$ and $\gamma_0 = 1.3 R_{comp}$ and $v_{rsh}$ are scaled as $R_{comp} \to (R_{comp} - 5.890) \times 10^3$ and $'_{rsh}$ $4 \times 10^{-6} v_{rsh}$. It is also observed that when inflow energy $\varepsilon$ is kept fixed, $R_m$ nonlinearly increases with Eddington rate. This is because, as $\varepsilon$ is kept fixed while $\dot{M}_{in}$ is varied, the amount of infalling energy converted to produce high energy protons is also fixed, so higher is the value of $\dot{M}_{in}$ (in the unit of Eddington rate), the larger is the distance of the shock surface (measured from the black hole) and the outflowing matter feels low inward gravitational pull, the result of which is the non-linear correlation of $R_m$ with $\dot{M}_{in}$.

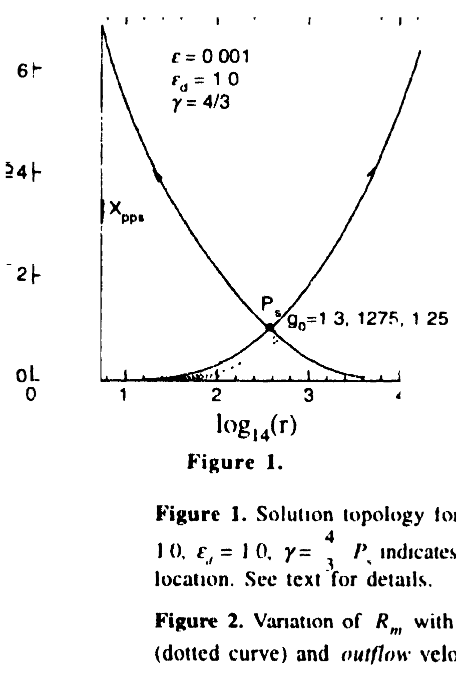


**Figure 1.**

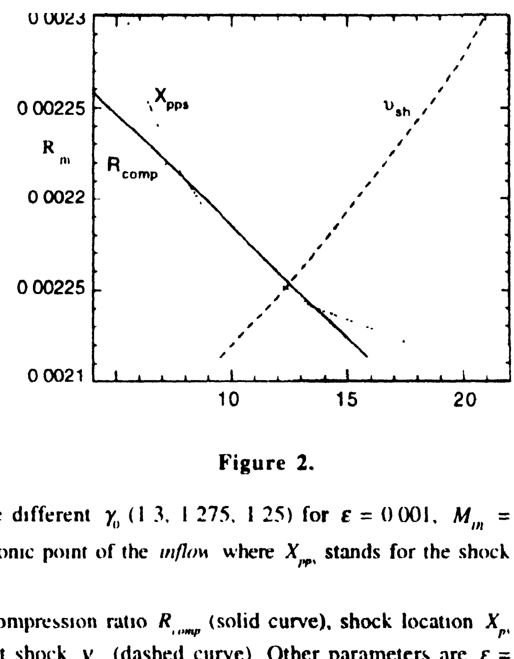


**Figure 2.**

**Figure 1.** Solution topology for three different $\gamma_0$ (1 3, 1 275, 1 25) for $\varepsilon$ = 0 001, $\dot{M}_{in}$ = 1 0, $\varepsilon_d$ = 1 0, $\gamma = \frac{4}{3}$ $P_s$ indicates the sonic point of the *inflow* where $X_{pps}$ stands for the shock location. See text for details.

**Figure 2.** Variation of $R_m$ with the compression ratio $R_{comp}$ (solid curve), shock location $X_{ps}$ (dotted curve) and *outflow* velocity at shock $v_{sh}$ (dashed curve) Other parameters are $\varepsilon$ = 0 001, $\dot{M}_{in}$ = 1 0 (In the unit of Eddington rate), $\gamma = \frac{4}{3}$ and $\gamma_0$ = 1 3

To have a better insight of the behavior of the outflow, we also studied the variation of $R_m$ as a function of the polytropic index of the inflow $\gamma$ and that of the outflow $\gamma_0$ for fixed $\varepsilon = 0.001$ and $\dot{M}_{in} = 1.0\varepsilon_d$. The range of $\gamma$ studied here are the range for which shock forms for the specified $\varepsilon$ and $\dot{M}_{in}$. The general observation is that $R_m$ correlates with $\gamma_0$. This is because as $\gamma_0$ increases, shock location and post shock density of matter does not change (as $\gamma_0$ does not have any role in shock formation or in determining the $R_{comp}$) but the sonic point of the *outflow* is pushed inward, hence the velocity with which outflow leaves the shock surface goes up resulting the increment in $R_{\dot{m}}$. However, it is observed that $R_{\dot{m}}$ anticorrelates with $\gamma$.

The basic conclusions of this work are the followings :

1. It is possible that outflows for quasi-spherical Bondi type accretion onto a Schwarzschild black hole are coming from the pair plasma pressure mediated shock surface.

2. The outflow rate monotonically increases with the specific energy of the inflow and nonlinearly increases with the Eddington rate of the infalling matter.

3. $R_m$, in general, correlates with $\gamma_0$ but anticorrelates with $\gamma$.

4. Generally speaking, as our model deals with high shock Mach number (low energy accretion) solutions, outflows in our work always generate from the supersonic branch of the inflow, *i.e.* shock is always located *inside* the sonic point.

5. Unlike the mass outflow from the *accretion disks* around black holes [2,3,4] here we found that the value of $R_m$ is distinguishably small. This is because matter is ejected out due to the pressure of the relativistic plasma pairs which is *less enough* in comparison to the pressure generated due to the presence of significant angular momentum However, in the present work we have dealt with only high Mach number solution which means matter is accreting with very low energy (cold inflow, as it is described in literature) This is another possible reason to obtain a low mass loss rate. If, instead of high Mach number solution, we would use low Mach number solution, *e.g.* high energy accretion, the mass outflow would be considerably higher (this is obvious because it has already been established in present work that $R_m$ increase with $\varepsilon$. In our next work, we will present this type of model by calculating $R_{comp}$ and $\left(\dfrac{\delta}{\varepsilon_1}\right)$ for low Mach number solution

So far, we made the computation around Schwarzschild black hole. Our work could be extended to study mass outflow in Kerr space time using pseudo-Kerr potential [14] and incorporating the frame dragging effect. This is under preparation and will be presented elsewhere.

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